Last week we discussed factorials – how we can take something common when we have factorials in some equations. Today let's discuss a couple of questions based on factorials. They look intimidating but they are pretty simple. Factorial is all about multiplication and hence there is a high probability that you will be able to take something common and cancel something. These techniques reduce our work significantly. Hence, seeing a factorial in a question should bring a smile to your face!

Question 1: Given that x, y and z are positive integers, is y!/x! an integer?

Statement 1: (x + y)(x – y) = z! + 1

Statement 2: x + y = 121

Solution: First let's focus on what the question is asking.

Is y!/x! an integer? When will division of two factorials lead to an integer? Take examples to understand 5!/4! = 5 (an integer).

3!/5! is not an integer.

10!/5! = 6*7*8*9*10 (an integer)

We can see that if y is greater than or equal to x, y!/x! will be an integer. If y is less than x, then we will obtain a proper fraction of the form 1/n. Let's go on to the statements now.

Statement 1: (x + y)(x – y) = z! + 1

x and y are positive integers. This mean (x + y) must be positive.

z is a positive integer so z! must be positive too. This means the right hand side must be positive. So the left hand side should be positive too. This means (x – y) must be positive i.e. x – y > 0 or x > y.

If x > y, y!/x! will not be an integer (as discussed above).
This statement alone is sufficient to answer the question.

Statement 2: x + y = 121

This statement doesn't tell us whether x is greater than y. Hence, this statement alone is not sufficient to answer the question.

Answer (A)

Question 2: Given that k = [(17!)^16 - (17!)^8] / [(17!)^8 + (17!)^4], what is the units digit of k/(17!)^4

(A) 0
(B) 1
(C) 3
(D) 5
(E) 9

Solution: The given expression can be easily made manageable. (I would suggest you to write it down on paper since the tons of brackets and operators used here make it difficult to understand.) All we have to remember is that the only thing that works with factorials is ‘taking common’

k = [(17!)^16 - (17!)^8] / [(17!)^8 + (17!)^4]
k = (17!)^8[(17!)^8 - 1] / [(17!)^4{(17!)^4 + 1}]
k = (17!)^4[(17!)^4 + 1] *[(17!)^4 - 1] / [(17!)^4 + 1]

(Using the identity a^2 – b^2 = (a – b)(a + b))

We get k = (17!)^4[(17!)^4 - 1]

Then, k/(17!)^4 = (17!)^4[(17!)^4 - 1]/(17!)^4

k/(17!)^4 = (17!)^4 – 1

What will be the unit’s digit of 17! ? To understand this, let’s talk about the concept of last digits of factorials now.

0! = 1
1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5040

Did you notice something about last digits? After 4!, all factorials have 0 as the last digit. Why? Because there is a 5 and a 2 in their calculation. So they will always contain 10 and hence, end with 0.

This means 17! will end with a 0 and so will (17!)^4. Therefore, the last digit of (17!)^4 – 1 will be 9.

Answer (E)

Based on the last digit concept, try this question now:

Question 3: If M is a positive integer, what is the last digit of 1! + 2! + … + M!?

Statement 1: M is a multiple of 4.
Statement 2: (M^2 + 1)/5 is an odd integer.